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Supersymmetric codimension-two branes in six-dimensional gauged supergravity

Hyun Min Lee

Department of Physics, Carnegie Mellon University,
5000 Forbes Avenue, Pittsburgh, PA 15213, U.S.A.
E-mail: hmlee@andrew.cmu.edu

Antonios Papazoglou

Institute of Cosmology & Gravitation, University of Portsmouth,
Portsmouth PO1 2EG, U.K.
E-mail: antonios.papazoglou@port.ac.uk

Abstract: We consider the six-dimensional Salam-Sezgin supergravity in the presence of codimension-2 branes. In the case that the branes carry only tension, we provide a way to supersymmetrise them by adding appropriate localised Fayet-Iliopoulos terms and modifying accordingly the supersymmetry transformations. The resulting brane action has $\mathcal{N}=1$ supersymmetry (SUSY). We find the axisymmetric vacua of the system and show that one has unwarped background solutions with "football"-shaped extra dimensions which always respect $\mathcal{N}=1$ SUSY, in contrast with the non-supersymmetric brane action background. Finally, we generically find multiple zero modes of the gravitino in this background and discuss how one could obtain a single chiral zero mode present in the low energy spectrum.

Keywords: Field Theories in Higher Dimensions, Supersymmetry Breaking, Supergravity Models, Flux compactifications.

Contents

1. Introduction

For the last decade, it has been an intensive effort to incorporate gravity for solving the particle physics problems. Particularly, in higher dimensional models with branes where the Standard Model (SM) particles are confined [1], the mass scale hierarchies in the SM can be understood from geometric factors in extra dimensions. Moreover, for the minimal supersymmetric extension of the SM (MSSM), the SUSY flavor problem can be ameliorated by a geometrical separation of the hidden sector from the visible sector in extra dimensions, the so called sequestering mechanism [2, 3]. In this case, the anomaly mediation [2, 4] can be a dominant contribution[1] to the soft mass parameters in the MSSM. The supersymmetric embedding of the brane action in the 5D warped supergravity was studied in [8] and the extension of the analysis to the 6D flat supergravity has been done in [6].

Recently there has been a renewed interest into the 6D Salam-Sezgin supergravity [9], due to the findings of the new warped solutions [10 − 14]. The warped background has the extra dimensions "spontaneously" compactified by $\mathrm{U}(1)_R$ flux on the warped product

[1]The Kähler potential is not of a sequestered form in higher than five dimensions [5, 6] but some global symmetry that is not broken by the messenger sector can keep the sequestering [7].

of the 4D Minkowski space and a deformed sphere (or general two-dimensional compact Riemann surfaces). Moreover, the branes with nonzero tensions are accommodated at the conical singularities, without the need of cutting and pasting the extra dimension as in the 5D case. Since the 4D Minkowski space is present as a unique regular solution with maximal symmetry [10], the warped solution has a feature of self-tuning of the cosmological constant [15][2] (for a review, see [18]). There have been a lot of follow-up works on this model (as well as its non-SUSY analogue [19]), such as the perturbation analysis [20 − 22], the gravitino spectrum [23], cosmological de-Sitter or scaling solutions [24],[3] regularisation of the conical singularities [26, 27], cosmology on a regularised brane [28, 29], modulus stabilisation [30], the Casimir effect [31], the effective 4D theory using the gradient expansion [32], exact wave solutions [33], etc. In the literature, however, the branes are regarded as breaking SUSY explicitly at the scale of brane tensions.

In this paper, we consider the supersymmetrisation of the brane tension action in a way compatible with the bulk SUSY in 6D Salam-Sezgin supergravity. We find that a brane-localised Fayet-Ilioupolos (FI) term[4] proportional to each brane tension must be introduced to cancel the SUSY variation of the brane tension term. With a nonzero FI term, we should also add in the action the brane-localised bilinear fermion terms that couple to the $\mathrm{U}(1)_R$ field strength. Furthermore, we should modify the SUSY transformation of the $\mathrm{U}(1)_R$ gaugino with a singular term. The Z_2 orbifold boundary conditions on the branes are also required to project out half of the bulk SUSY.

Consequently, solving the modified equations of motion with singular FI terms, we find that the axisymmetric warped solution of the non-SUSY brane action is maintained, because the localised FI term is cancelled by a singular piece of the $\mathrm{U}(1)_R$ field strength. However, the Wilson line phase of the gauge potential is now fixed to be nonzero at the brane position due to the extra singular term in the gauge field equation. From the SUSY variations of the spinors, we show that the only supersymmetric solution with branes is the unwarped "football"-shaped compactification. Furthermore, we find that the FI terms can affect the number of zero modes of gravitino and we expect that the same is true for any $\mathrm{U}(1)_R$ charged bulk field.

By analysing the equation for the 4D component gravitino, we show that even after the Z_2 projection around the branes, there are generically multiple normalizable zero modes of the gravitino. In particular, for the "football" solutions, there are multiple chiral zero modes only from the left-handed gravitino: the one with zero winding number and pairs of chiral zero modes with nonzero winding numbers $(m, -m)$. The mass terms for them would be forbidden unless the two $\mathrm{U}(1)$ gauge symmetries in the system, the $\mathrm{U}(1)_Q$ isometry of the axisymmetric extra dimensions and the $\mathrm{U}(1)_R$ symmetry, are broken. In this "football" case, we propose that it is possible to have only one chiral zero mode of the 4D gravitino left (with zero winding number), if a linear combination of the $\mathrm{U}(1)$ symmetries remains

[2]See, however, refs. [16, 17].

[3]See ref. [25] for old cosmological solutions without the presence of branes.

[4]An arbitrary brane-localised FI term was considered to see the effect on the quantization condition in refs. [11, 17]. In 6D global SUSY, the effect of the FI term on the localisation and the Kaluza-Klein(KK) mass spectrum of bulk fields was discussed in ref. [34].

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unbroken at low energies. The survival of only one chiral gravitino would be what one should expect from 4D unbroken $\mathcal{N} = 1$ supergravity.

The paper is organized as follows. First we present the bulk action of the 6D Salam-Sezgin supergravity to fix the notations. Then we consider the supersymmetrisation of the brane tension action and derive the required supersymmetric brane-bulk couplings. We go on to discuss the modified solutions with the localized FI terms, identify the supersymmetric football-shaped solution and study the effect on the zero modes of gravitino. Finally, the conclusions are drawn.

2. Six-dimensional Salam-Sezgin supergravity

The six-dimensional Salam-Sezgin supergravity [9] consists of gravity coupled to a dilaton field ϕ, a $\mathrm{U}(1)_R$ gauge field A_M and a Kalb-Ramond field B_{MN}, along with the necessary SUSY fermionic fields, the gravitino ψ_M, the dilatino χ and the gaugino λ where all spinors are 6D Weyl. The $\mathrm{U}(1)_R$ gauge field corresponds to the gauging of the R-symmetry of six-dimensional supergravity. The complete bulk Langrangian up to four fermion terms is given by

$$\begin{aligned}
e_6^{-1}\mathcal{L}_{\mathrm{bulk}} = {} & R - \frac{1}{4}(\partial_M\phi)^2 - \frac{1}{12}e^{\phi}G_{\mathrm{MNP}}G^{\mathrm{MNP}} - \frac{1}{4}e^{\frac{1}{2}\phi}F_{\mathrm{MN}}F^{\mathrm{MN}} - 8g^2e^{-\frac{1}{2}\phi} \\
& +\bar\psi_M\Gamma^{\mathrm{MNP}}\mathcal{D}_N\psi_P + \bar\chi\Gamma^M\mathcal{D}_M\chi + \bar\lambda\Gamma^M\mathcal{D}_M\lambda \\
& +\frac{1}{4}\left(\partial_M\phi\right)\left(\bar\psi_N\Gamma^M\Gamma^N\chi + \bar\chi\Gamma^N\Gamma^M\psi_N\right) \\
& +\frac{1}{24}e^{\frac{1}{2}\phi}G_{\mathrm{MNP}}\big(\bar\psi^R\Gamma_{[R}\Gamma^{\mathrm{MNP}}\Gamma_{S]}\psi^S + \bar\psi_R\Gamma^{\mathrm{MNP}}\Gamma^R\chi \\
& \qquad -\bar\chi\Gamma^R\Gamma^{\mathrm{MNP}}\psi_R - \bar\chi\Gamma^{\mathrm{MNP}}\chi + \bar\lambda\Gamma^{\mathrm{MNP}}\lambda\big) \\
& -\frac{1}{4\sqrt{2}}e^{\frac{1}{4}\phi}F_{\mathrm{MN}}\left(\bar\psi_Q\Gamma^{\mathrm{MN}}\Gamma^Q\lambda + \bar\lambda\Gamma^Q\Gamma^{\mathrm{MN}}\psi_Q + \bar\chi\Gamma^{\mathrm{MN}}\lambda - \bar\lambda\Gamma^{\mathrm{MN}}\chi\right) \\
& +i\sqrt{2}ge^{-\frac{1}{4}\phi}\left(\bar\psi_M\Gamma^M\lambda + \bar\lambda\Gamma^M\psi_M - \bar\chi\lambda + \bar\lambda\chi\right).
\end{aligned} \tag{2.1}$$

The field strengths of the gauge and the Kalb-Ramond fields are defined as

$$F_{\mathrm{MN}} = \partial_M A_N - \partial_N A_N, \tag{2.2}$$

$$G_{\mathrm{MNP}} = 3\partial_{[M}B_{NP]} + \frac{3}{2}F_{[MN}A_{P]}, \tag{2.3}$$

and satisfy the Bianchi identities

$$\partial_{[Q}F_{MN]} = 0, \tag{2.4}$$

$$\partial_{[Q}G_{MNP]} = \frac{3}{2}F_{\mathrm{QM}}F_{\mathrm{NP}}. \tag{2.5}$$

For $\delta A_M = \partial_M\Lambda$ under the $\mathrm{U}(1)_R$, the Kalb-Ramond field B_{MN} transforms as

$$\delta B_{\mathrm{MN}} = -\Lambda F_{\mathrm{MN}}. \tag{2.6}$$

All the spinors have the same charge normalized to $+1$ under $\mathrm{U}(1)_R$, so the covariant derivative of the gravitino, for instance, is given by

$$\mathcal{D}_M\psi_N = \left(\partial_M + \frac{1}{4}\omega_{\mathrm{MAB}}\Gamma^{\mathrm{AB}} - igA_M\right)\psi_N. \tag{2.7}$$

The action for this Lagrangian is invariant under the following local $\mathcal{N} = 2$ SUSY transformations (up to the trilinear fermion terms):

$$\begin{aligned}
\delta e_M^A &= \frac{1}{4}\left(-\bar{\varepsilon}\Gamma^A\psi_M + \bar{\psi}_M\Gamma^A\varepsilon\right), && (2.8)\\
\delta\phi &= \frac{1}{2}(\bar{\varepsilon}\chi + \bar{\chi}\varepsilon), && (2.9)\\
\delta B_{\mathrm{MN}} &= A_{[M}\delta A_{N]} + \frac{1}{4}e^{-\frac{1}{2}\phi}\big(\bar{\varepsilon}\Gamma_M\psi_N - \bar{\psi}_N\Gamma_M\varepsilon - \bar{\varepsilon}\Gamma_N\psi_M + \bar{\psi}_M\Gamma_N\varepsilon\\
&\qquad\qquad + \bar{\varepsilon}\Gamma_{\mathrm{MN}}\chi - \bar{\chi}\Gamma_{\mathrm{MN}}\varepsilon\big), && (2.10)\\
\delta\chi &= -\frac{1}{4}(\partial_M\phi)\Gamma^M\varepsilon + \frac{1}{24}e^{\frac{1}{2}\phi}G_{\mathrm{MNP}}\Gamma^{\mathrm{MNP}}\varepsilon, && (2.11)\\
\delta\psi_M &= \mathcal{D}_M\varepsilon + \frac{1}{48}e^{\frac{1}{2}\phi}G_{\mathrm{PQR}}\Gamma^{\mathrm{PQR}}\Gamma_M\varepsilon, && (2.12)\\
\delta A_M &= \frac{1}{2\sqrt{2}}e^{-\frac{1}{4}\phi}(\bar{\varepsilon}\Gamma_M\lambda - \bar{\lambda}\Gamma_M\varepsilon), && (2.13)\\
\delta\lambda &= \frac{1}{4\sqrt{2}}e^{\frac{1}{4}\phi}F_{\mathrm{MN}}\Gamma^{\mathrm{MN}}\varepsilon - i\sqrt{2}g\,e^{-\frac{1}{4}\phi}\varepsilon. && (2.14)
\end{aligned}$$

The above spinors are chiral with handednesses

$$\Gamma^7\psi_M = +\psi_M, \quad \Gamma^7\chi = -\chi, \quad \Gamma^7\lambda_1 = +\lambda_1, \quad \Gamma^7\varepsilon = +\varepsilon. \tag{2.15}$$

Taking into account that $\Gamma^7 = \sigma^3 \otimes \mathbf{1}$ (see appendix A), the 6D (8-component) spinors can be decomposed to 6D Weyl (4-component) spinors as

$$\psi_M = (\tilde{\psi}_M, 0)^T, \quad \chi = (0, \tilde{\chi})^T, \quad \lambda = (\tilde{\lambda}, 0)^T, \quad \varepsilon = (\tilde{\varepsilon}, 0)^T. \tag{2.16}$$

For later use, we decompose the 6D Weyl spinor $\tilde{\psi}$ to $\tilde{\psi} = (\tilde{\psi}_L, \tilde{\psi}_R)^T$, satisfying $\gamma^5(\tilde{\psi}_L, 0)^T = +(\tilde{\psi}_L, 0)^T$ and $\gamma^5(0, \tilde{\psi}_R)^T = -(0, \tilde{\psi}_R)^T$.

3. Supersymmetrising the brane tension action

In this section, we will add in the previous action codimension-two branes with nonzero tension. With this addition, the total action is no longer invariant under the transformations (2.8)-(2.14). We will, thus, modify our action and SUSY transformations, so that the brane-bulk system is rendered supersymmetric. With the modification that we propose, we show that the bulk action remains supersymmetric while the brane action preserves $\mathcal{N} = 1$ SUSY.

3.1 Requirements for the supersymmetric brane action

Let us add to the bulk Lagrangian a term for a brane located at the position $y = y_i$, where y is the internal space 2D coordinate. This brane Lagrangian will be given by

$$\mathcal{L}_{\mathrm{brane}} = -e_4 T_i \delta^{(2)}(y - y_i), \tag{3.1}$$

where T_i is the brane tension and the $2D$ delta function is defined as $\int d^2y\delta^{(2)}(y-y_i)=1$.

The SUSY transformation of the brane action is non-vanishing as follows,

$$\delta\mathcal{L}_{\rm brane}=-e_4\frac{1}{4}T_i\delta^{(2)}(y-y_i)(\bar\psi_\mu\Gamma^\mu\varepsilon+{\rm h.c.}). \tag{3.2}$$

On the other hand, because the gravitino is charged under $\mathrm{U}(1)_R$, varying the gravitino kinetic term under (2.12), it contains a piece of the gauge field strength as

$$\begin{aligned}\delta\mathcal{L}_{\rm gravitino}&\supset e_6\bar\psi_M\Gamma^{\rm MNP}\mathcal{D}_N\mathcal{D}_P\varepsilon\\&=-\frac{i}{2}e_6g\bar\psi_M\Gamma^{\rm MNP}\varepsilon F_{\rm NP}+\cdots\,.\end{aligned} \tag{3.3}$$

We can utilise the above term of the gravitino vatiation to cancel the brane tension term as following. The $\mathrm{U}(1)_R$ field can have in principle FI localised terms [11, 17] parameterized by constants ξ_i. We can then define a hatted field strength $\hat F_{\rm MN}$

$$\hat F_{\mu\nu}=F_{\mu\nu},\quad \hat F_{\mu m}=F_{\mu m}, \tag{3.4}$$

$$\hat F_{mn}=F_{mn}-\epsilon_{mn}\xi_i\,\frac{\delta^{(2)}(y-y_i)}{e_2}, \tag{3.5}$$

where ϵ_{mn} is the 2D volume form, and rewrite the variation of the gravitino kinetic term as

$$\begin{aligned}\delta\mathcal{L}_{\rm gravitino}\supset&-\frac{i}{2}e_6g\bar\psi_M\Gamma^{\rm MNP}\varepsilon\hat F_{\rm NP}\\&+e_4g\xi_i\delta^{(2)}(y-y_i)\bar\psi_\mu\Gamma^\mu\gamma^5\varepsilon+\cdots\,,\end{aligned} \tag{3.6}$$

where use is made of $\Gamma^{mn}\epsilon_{mn}=2\Gamma^{56}=2i\sigma^3\otimes\gamma^5$, the 6D chirality condition, $\sigma^3\otimes\mathbf{1}\varepsilon=\varepsilon$, and $\frac{e_6}{e_2}=e_4$. Then, the first term cancels the variation of the bulk fermion bilinear term, if the $F_{\rm MN}$ in the fermion bilinear term is replaced with $\hat F_{\rm MN}$. Most importantly, the second term has the right form to cancel the variation of the brane tension term. The condition for this to happen is that,

$$\left(\gamma_5-\frac{T_i}{4g\xi_i}\right)\varepsilon(y_i)=0. \tag{3.7}$$

In other words, decomposing the SUSY variation spinor as $\varepsilon=(\tilde\varepsilon,0)^T$ with $\tilde\varepsilon=(\tilde\varepsilon_L,\tilde\varepsilon_R)^T$, the following should be satisfied,

$$\left(1-\frac{T_i}{4g\xi_i}\right)\tilde\varepsilon_L(y_i)=0, \tag{3.8}$$

$$\left(1+\frac{T_i}{4g\xi_i}\right)\tilde\varepsilon_R(y_i)=0. \tag{3.9}$$

Thus, fixing the FI terms with the brane tensions as $\xi_i=\frac{T_i}{4g}$ or $-\frac{T_i}{4g}$, one needs to impose that either $\tilde\varepsilon_R$ or $\tilde\varepsilon_L$ vanish on the brane. Therefore, only $\mathcal{N}=1$ SUSY can be preserved on the brane. For other values of ξ_i, both $\tilde\varepsilon_L$ and $\tilde\varepsilon_R$ must vanish at the brane, so there would be no SUSY left. Furthermore, when $F_{\rm MN}$ is replaced by $\hat F_{\rm MN}$ in both the bulk

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action and the SUSY transformations, keeping the form of terms containing $G_{\rm MNP}$ and A_M to be the same[5] as in the case with no branes, the modified bulk action is supersymmetric up to four fermion terms.

From now on, we choose $\xi_i = \frac{T_i}{4g}$ for all branes[6] present in the internal space, so that there is $\mathcal{N}=1$ SUSY remaining in the brane action with a SUSY parameter $\tilde{\varepsilon}_L$ non vanishing on the branes. This choice is made to agree with the no-brane Salam-Sezgin vacuum [9] where a constant $\tilde{\varepsilon}_L$ is a Killing spinor.

3.2 Orbifold boundary conditions

Once an FI term has been chosen to make the brane tension action invariant under the SUSY transformations, one has in addition to impose that $\tilde{\varepsilon}_R$ vanishes at the brane position to preserve $\mathcal{N}=1$ SUSY on the brane. This can be easily accomplished if we assume an orbifold Z_2 symmetry around the brane.

If the local complex coordinate around the brane is z (in locally polar coordinates $z = re^{i\theta}$), then the Z_2 symmetry corresponds to

$$z \leftrightarrow -z \qquad (\text{or} \;\; \theta \leftrightarrow \theta + \pi). \tag{3.10}$$

The same Z_2 was also introduced in [21] to avoid the possible instability of a negative tension brane. We should then assign Z_2 parities to all bulk fields and, of course, the SUSY variation parameters $\tilde{\varepsilon}_L$ and $\tilde{\varepsilon}_R$. A consistent choice of parities for the fields and the SUSY variation parameter is

$$\text{even}: \;\; \tilde{\psi}_{\alpha L},\; \tilde{\psi}_{aR},\; \tilde{\lambda}_L,\; \tilde{\chi}_R,\; \tilde{\varepsilon}_L,\; A_\alpha,\; B_{\alpha\beta},\; B_{ab},\; \phi, \tag{3.11}$$
$$\text{odd}: \;\; \tilde{\psi}_{\alpha R},\; \tilde{\psi}_{aL},\; \tilde{\lambda}_R,\; \tilde{\chi}_L,\; \tilde{\varepsilon}_R,\; A_a,\; B_{\alpha a}. \tag{3.12}$$

where the gauge field, the Kalb-Ramond field and the gravitino have been written with locally flat indices, *e.g.,* $A_A = e_A^{\ M} A_M$, so that the parity assignments do not depend on the coordinate system. It is obvious that the above choice of parities forces $\tilde{\varepsilon}_R$ to vanish on the brane position.

In the case with two branes system, the warped vacua of [10] have an axially symmetric internal space. The above Z_2 symmetry about both branes present, is just a discrete subgroup of the axial symmetry. On the other hand, for the general warped solutions with multiple branes [13], we require the holomorphic function $V(z)$ in the metric to satisfy the condition $|V(-z+z_i)| = |V(z-z_i)|$, where z_i is the i-th brane position.

3.3 The supersymmetric brane-bulk coupling

As a consequence of introducing the localised FI terms, we have seen that the brane tension action is made compatible with the bulk SUSY transformations. The supersymmetric

[5] We note, however, that the *solutions* for the gauge field and the Kalb-Ramond field can be changed due to the singular FI term compared to the case with no branes, as will be shown later.

[6] When there are different FI terms on the branes, there is no SUSY left, which corresponds to an explicit SUSY breaking by orbifolding.

action of the brane-bulk system up to four fermion terms is

$$
\begin{aligned}
e_6^{-1}\mathcal{L}_{\rm SUSY} = {} & R - \frac{1}{4}(\partial_M\phi)^2 - \frac{1}{12}e^{\phi}G_{\rm MNP}G^{\rm MNP} - \frac{1}{4}e^{\frac{1}{2}\phi}\hat{F}_{\rm MN}\hat{F}^{\rm MN} - 8g^2 e^{-\frac{1}{2}\phi} \\
& +\bar{\psi}_M\Gamma^{\rm MNP}\mathcal{D}_N\psi_P + \bar{\chi}\Gamma^M\mathcal{D}_M\chi + \bar{\lambda}\Gamma^M\mathcal{D}_M\lambda \\
& +\frac{1}{4}(\partial_M\phi)(\bar{\psi}_N\Gamma^M\Gamma^N\chi + \bar{\chi}\Gamma^N\Gamma^M\psi_N) \\
& +\frac{1}{24}e^{\frac{1}{2}\phi}G_{\rm MNP}\big(\bar{\psi}^R\Gamma_{[R}\Gamma^{\rm MNP}\Gamma_{S]}\psi^S + \bar{\psi}_R\Gamma^{\rm MNP}\Gamma^R\chi \\
& \qquad\qquad -\bar{\chi}\Gamma^R\Gamma^{\rm MNP}\psi_R - \bar{\chi}\Gamma^{\rm MNP}\chi + \bar{\lambda}\Gamma^{\rm MNP}\lambda\big) \\
& -\frac{1}{4\sqrt{2}}e^{\frac{1}{4}\phi}\hat{F}_{\rm MN}\left(\bar{\psi}_Q\Gamma^{\rm MN}\Gamma^Q\lambda + \bar{\lambda}\Gamma^Q\Gamma^{\rm MN}\psi_Q + \bar{\chi}\Gamma^{\rm MN}\lambda - \bar{\lambda}\Gamma^{\rm MN}\chi\right) \\
& +i\sqrt{2}ge^{-\frac{1}{4}\phi}\left(\bar{\psi}_M\Gamma^M\lambda + \bar{\lambda}\Gamma^M\psi_M - \bar{\chi}\lambda + \bar{\lambda}\chi\right) \\
& -\frac{e_4}{e_6}T_i\delta^{(2)}(y-y_i),
\end{aligned}
\tag{3.13}
$$

where the modified gauge field strength is

$$
\hat{F}_{\rm MN} = F_{\rm MN} - \delta^m_M\delta^n_N\epsilon_{\rm mn}\xi_i\frac{\delta^{(2)}(y-y_i)}{e_2}, \tag{3.14}
$$

with

$$
\xi_i = \frac{T_i}{4g}. \tag{3.15}
$$

Here $G_{\rm MNP}$ is the same as eq. (2.3). The SUSY transformation of λ is modified as

$$
\delta\lambda = \frac{1}{4\sqrt{2}}e^{\frac{1}{4}\phi}\hat{F}_{\rm MN}\Gamma^{\rm MN}\varepsilon - i\sqrt{2}g\,e^{-\frac{1}{4}\phi}\varepsilon, \tag{3.16}
$$

but the SUSY transformations for the other fields are the same as eqs. (2.8)–(2.13). The important ingredient of the above modifications is that we have a brane term linear in $F_{\rm MN}$, the brane-localised FI term. In other words, there is a brane coupling to the magnetic flux, which is proportional to the brane tension. We note that the modified gauge field strength satisfies the Bianchi identity $\partial_{[Q}\hat{F}_{MN]} = 0$ even with the singular FI term.

One could be worried by the squared terms of the two-dimensional delta functions appearing in the kinetic term $\hat{F}_{\rm MN}\hat{F}^{\rm MN}$. However, SUSY requires these terms to be present and are a usual ingredient of orbifold supersymmetric theories [35, 34]. The delta squared terms, *i.e.,* $\delta^2(0)$, appear naturally in orbifolds, when bulk and brane fields are coupled supersymmetrically. One can obtain the same form $\hat{F}_{\rm MN}\hat{F}^{\rm MN}$ in a 6D off-shell supersymmetric U(1) theory on T^2/Z_2, after the auxiliary field of the bulk vector multiplet is eliminated [34]. It has been known that the $\delta^2(0)$ term provides counterterms, which are necessary to maintain supersymmetry in explicit calculations on orbifolds, like the scattering amplitude and the self-energy correction for a brane field [35]. In our case, we have not introduced brane multiplets other than the tension. The case with brane multiplets will be studied elsewhere so the usual discussion on the $\delta^2(0)$ term on orbifolds is expected to hold.

As will be shown in the next section, when one looks for the solutions of the equations of motion of the above system, the singular term in the modified gauge field strength is cancelled by the singular part of the background value of $F_{\rm MN}$, without affecting the solution of the metric and the dilaton obtained for the non-SUSY brane action. Only the linear term in $\hat{F}_{\rm MN}$ with arbitrary coefficient has been considered for the non-SUSY brane action [11, 17]. However, in this case, even if $F_{\rm MN}$ acquires a singular piece to satisfy the gauge field equation, it would lead to a problematic two-dimensional delta squared term in the Einstein and dilaton equations of motion [17]. Moreover, when one looks at the low energy effective theory, there is a worrisome singular delta squared term corresponding to the mass term of 4D $\mathrm{U}(1)_R$ gauge boson A_μ from $G_{\mu mn}G^{\mu mn}$. However, by solving the linearized equation for $B_{\rm MN}$ and inserting the solution for $B_{\mu m}$ into the action, the singular piece of the $B_{\mu m}$ cancels the contribution of the FI term in $G_{\mu mn}$, ending up with the regular action where the gauge boson gets a finite mass from the FI terms. Similar cancellations happen in 5D [36] and 6D [6] supergravities coupled to branes.

There are some known anomaly-free models including the non-abelian gauge fields in 6D gauged supergravity [37, 38]. In these cases, an abelian flux can be also turned on in the direction of the non-abelian gauge fields. For instance, in the model with $E_7 \times E_6 \times \mathrm{U}(1)_R$ with hyperino $(\mathbf{912}, \mathbf{0})_{\mathbf{0}}$, the $\mathrm{U}(1)$ contained in E_6 can also develop a nonzero flux, still maintaining the warped solution that was obtained for the Salam-Sezgin supergravity [23]. As a result, E_6 is broken down to $\mathrm{SO}(10)$ in the bulk and the adjoint fermions of E_6 can survive as two chiral **16**'s of $\mathrm{SO}(10)$ [37]. Even in this more general case, the supersymmetric brane action obtained for the Salam-Sezgin supergravity remains the same.

Furthermore, we can always introduce arbitrary localized FI terms for any abelian factor[7] of the bulk gauge group other than $\mathrm{U}(1)_R$ in a supersymmetric way because there is no constraint from the variation of the gravitino kinetic term unlike eq. (3.6). We only have to modify the gauge field strength appearing in both the bulk action and the SUSY transformation of the corresponding gaugino like in eqs. (3.14) and (3.16), except the term included in $G_{\rm MNP}$. Thus, it is straightforward to see that the localised FI terms generated in 6D global SUSY case [34] are embedded into a supergravity theory.

4. Modification of the background solution due to the SUSY-brane action

In the present section, we will study the effect of the brane-localized FI terms to the warped axisymmetric solution that was obtained for non-SUSY brane action. We will see that the geometry is not modified by the latter addition, but the gauge field solution and the quantization condition change.

4.1 The modified equations of motion

We will study vacua where the Kalb-Ramond field is consistently (*i.e.,* satisfying its equation of motion) set to zero. Then, the Einstein equations derived from the modified ac-

[7]This does not include $\mathrm{U}(1)$ directions of non-abelian groups, as the one in E_6 mentioned above.

tion (3.13) are

$$\begin{aligned} R_{\mathrm{MN}} = &\ 2g^2\, e^{-\frac{1}{2}\phi} g_{\mathrm{MN}} + \frac{1}{2} e^{\frac{1}{2}\phi} \left(\hat{F}_{\mathrm{MP}} \hat{F}_{N}{}^{P} - \frac{1}{8} g_{\mathrm{MN}} \hat{F}^2_{\mathrm{PQ}} \right) \\ & + \frac{1}{4} \partial_M \phi \partial_N \phi + T^i_{\mathrm{MN}} \,, \end{aligned} \tag{4.1}$$

where $T^i_{\mathrm{MN}} = -\frac{1}{2}\frac{\sqrt{g_4}}{\sqrt{g_6}} T_i \left(g^4_{\mu\nu} \delta^\mu_M \delta^\nu_N - g_{\mathrm{MN}} \right) \delta^{(2)}(y - y_i)$ is the brane tension contribution (with $g^4_{\mu\nu}$ the 4D induced metric). Furthermore, the dilaton and the gauge field equations read

$$\Box^{(6)} \phi = \frac{1}{4} e^{\frac{1}{2}\phi} \hat{F}^2_{\mathrm{PQ}} - 8g^2\, e^{-\frac{1}{2}\phi} \,, \tag{4.2}$$

$$\partial_M \left(\sqrt{-g} e^{\frac{1}{2}\phi} \hat{F}^{\mathrm{MN}} \right) = 0 \,. \tag{4.3}$$

4.2 The modified warped solution

Assuming axial symmetry in the internal space, the form of the general warped solution of [10 – 12] is maintained, except that the solution for F_{mn} is being replaced with the hatted one. Thus, the metric, the gauge field and the dilaton solutions are respectively

$$ds^2 = W^2(r) \eta_{\mu\nu} dx^\mu dx^\nu + R^2(r) \Big(dr^2 + \lambda^2 \Theta^2(r) d\theta^2 \Big), \tag{4.4}$$

$$\hat{F}_{r\theta} = \lambda q \frac{\Theta R^2}{W^6}, \tag{4.5}$$

$$\phi = 4 \ln W, \tag{4.6}$$

with

$$R = \frac{W}{f_0}, \qquad \Theta = \frac{r}{W^4}, \tag{4.7}$$

$$W^4 = \frac{f_1}{f_0}, \qquad f_0 = 1 + \frac{r^2}{r_0^2}, \qquad f_1 = 1 + \frac{r^2}{r_1^2}, \tag{4.8}$$

where q is a constant denoting the magnetic flux, and the two radii r_0, r_1 are given by

$$r_0^2 = \frac{1}{2g^2}, \quad r_1^2 = \frac{8}{q^2}. \tag{4.9}$$

In the warped solution, the metric has two conical singularities, one at $r = 0$ and the other at $r = \infty$, which is at finite proper distance from the former one. The deficit angles δ_i of these singularities (supported by brane tensions $T_i = 2\delta_i$) are given by

$$\frac{\delta_0}{2\pi} = 1 - \lambda, \tag{4.10}$$

$$\frac{\delta_\infty}{2\pi} = 1 - \lambda \frac{r_1^2}{r_0^2}. \tag{4.11}$$

In the unwarped limit, *i.e.*, for $r_0 = r_1$, the two brane tensions must be equal.

Writing the delta function in eq. (3.14) in polar coordinates around $r=0$ as $\delta^{(2)}(y-y_i)/e_2 = \delta(r)/(2\lambda\pi r)$ and $\epsilon_{r\theta} = \lambda r$, eq. (4.5) becomes

$$F_{r\theta} - \frac{\xi_0}{2\pi}\delta(r) = \lambda q \frac{\Theta R^2}{W^6}. \tag{4.12}$$

Then, applying Stokes theorem around the patch including $r=0$, one obtains that $A_\theta(0) = \xi_0/(2\pi)$ and thus the solution of the only non-zero component of the gauge field is

$$A_\theta = -\frac{4\lambda}{q}\left(\frac{1}{f_1} - 1\right) + \frac{\xi_0}{2\pi}. \tag{4.13}$$

Likewise, the gauge potential in the patch surrounding $r=\infty$ is

$$A_\theta = -\frac{4\lambda}{q}\frac{1}{f_1} + \frac{\xi_\infty}{2\pi}. \tag{4.14}$$

Hence, after connecting the gauge field solutions in two patches by a gauge transformation and requiring that it is single valued under 2π rotations, we find the following quantization condition should hold

$$\frac{4\lambda g}{q} = n + \frac{g}{2\pi}(\xi_\infty - \xi_0), \quad n \in \mathbf{Z}. \tag{4.15}$$

In other words, we find that the FI terms fix the Wilson line phases of the gauge potential to be non-vanishing on the branes and can contribute to the quantization condition for $\xi_0 \neq \xi_\infty$, *i.e.,* when $T_0 \neq T_\infty$. Since the covariant derivative has the same form as in the case with no branes, the modified background solution for the gauge potential changes the equations of motions of the other bulk fields and can affect the number of their zero modes. Using the flux quantization (4.15) with eqs. (4.10) and (4.11), we obtain the brane tensions are related as

$$\left(1 - \frac{T_0}{4\pi}\right)\left(1 - \frac{T_\infty}{4\pi}\right) = \left[n + \frac{g}{2\pi}(\xi_\infty - \xi_0)\right]^2. \tag{4.16}$$

5. Supersymmetry of the background solution

Calculating the fermionic SUSY variations (2.11), (2.12), (2.14) for the above background solution, we can find in which cases the background respects or breaks SUSY. In the general warped background, SUSY is completely broken in the bulk. This can be seen just from the SUSY transformation of the dilatino,

$$\delta\chi = -\frac{W'}{W}\left[\cos\theta\sigma^1 \otimes \gamma^5 + \sin\theta\sigma^2 \otimes \mathbf{1}\right]\varepsilon, \tag{5.1}$$

which is always non-zero. In the special case of zero warping, *i.e.,* when $W' = 0$, we need to study the remaining SUSY transformations.

When there is no brane present, the solution (4.4) becomes a sphere compactification, known as the Salam-Sezgin vacuum [9]. The nontrivial SUSY transformations of the fermions are

$$\delta\lambda = i\sqrt{2}g(\gamma^5 - 1)\varepsilon, \tag{5.2}$$

$$\delta\psi_\theta = \left[\partial_\theta + i\Big(1 - \frac{1}{f_0}\Big)(\gamma^5 - 1)\right]\varepsilon. \tag{5.3}$$

In this case, there exists a constant Killing spinor $\tilde{\varepsilon}_L$, which means that $\mathcal{N}=1$ SUSY is preserved.

For the "football"-shaped extra dimensions [15], there are two branes of equal tension, $T_0 = T_\infty$, located at the poles of the sphere. The warp factor is constant, so we have that $q = 4g$ and $\lambda = n$. Since $n > 1$, the space has angle excess and thus the brane tensions are negative[8] (see [21] for a discussion on negative tension branes). In this case, the FI terms make the gauge potential nonzero at the branes, but they do not contribute to the quantization condition in eq. (4.15). In the patch surrounding the brane at $r = 0$, the nontrivial fermionic SUSY transformations are

$$\delta\lambda = i\sqrt{2}g(\gamma^5 - 1)\varepsilon, \tag{5.4}$$

$$\begin{aligned}\delta\psi_\theta &= \left[\partial_\theta + \frac{i}{2}\left\{1 + n\left(1 - \frac{2}{f_0}\right)\right\}\gamma^5 + in\left(\frac{1}{f_0} - 1\right) - i\frac{g\xi_0}{2\pi}\right]\varepsilon \\ &= \left[\partial_\theta + \frac{i}{2}\left\{1 + n\left(1 - \frac{2}{f_0}\right)\right\}(\gamma^5 - 1)\right]\varepsilon,\end{aligned} \tag{5.5}$$

where use is made of $g\xi_0 = \frac{1}{4}T_0 = \pi(1-n)$ from eq. (4.10) in the last line. Then, for a non-zero left-handed variation parameter $\tilde{\varepsilon}_L$, for which the gaugino variation is manifestly zero, the remaining nonzero gravitino variation is

$$\delta\tilde{\psi}_{\theta L} = \partial_\theta \tilde{\varepsilon}_L. \tag{5.6}$$

So, there exists a constant Killing spinor $\tilde{\varepsilon}_L$ which is Z_2-even with respect to the $r = 0$ brane. Thus, we find that the modified spin connection are cancelled by the nonzero Wilson line phases at the brane positions, so that $\mathcal{N}=1$ SUSY is preserved for the football solution. This is to be compared with the case of non-SUSY brane action in [23], where only the case of odd monopole number n would allow for $\mathcal{N}=1$ SUSY on the brane.

6. The gravitino zero modes

As we have seen in section 4 and in particular in eqs. (4.13) and (4.14), there are in general two possible inequivalent Wilson line phases at the conical singularities due to the localized FI terms. In this section, we discuss the effect of these Wilson line phases to the existence of massless modes of the gravitino. We will also note the differences from the result obtained in the case for a non-SUSY brane action [23].

For comparison with our earlier work [23], let us move to a Gaussian normal coordinate system, where the warped solution is written as

$$ds^2 = W^2\eta_{\mu\nu}dx^\mu dx^\nu + d\rho^2 + a^2 d\theta^2, \tag{6.1}$$

with $d\rho = Rdr, a = \lambda R\Theta$.

[8] In view of that, we should have rather called the space "pumpkin"-shaped, however, we keep the term "football" for simplicity.

After decomposing the 4D vector part[9] of the 6D Weyl gravitino $\psi_\mu = (\tilde{\psi}_\mu, 0)^T$ as $\tilde{\psi}_\mu = (\tilde{\psi}_{\mu L}, \tilde{\psi}_{\mu R})^T$ in terms of the 4D Weyl spinors, we make a Fourier expansion of them as

$$\tilde{\psi}_{\mu L} = \sum_m \tilde{\psi}^{(m)}_{\mu L}(x)\varphi^{(m)}_L(\rho)e^{im\theta}, \tag{6.2}$$

$$\tilde{\psi}_{\mu R} = \sum_m \tilde{\psi}^{(m)}_{\mu R}(x)\varphi^{(m)}_R(\rho)e^{im\theta}. \tag{6.3}$$

By the redefinition of the 4D gravitino, there is no mixing of $\tilde{\psi}_\mu$ with the other fermionic modes [23]. To obtain the massless modes, we set $\bar{\sigma}^\alpha\partial_\alpha\tilde{\psi}^{(m)}_{\mu L} = \sigma^\beta\partial_\beta\tilde{\psi}^{(m)}_{\mu R} = 0$. Then, the equations of left-handed and right-handed gravitinos are decoupled [23] and read

$$\left[\partial_\rho + \frac{W'}{W} + \frac{1}{a}\left(m - \frac{1}{2}\omega - gA_\theta\right)\right]\varphi^{(m)}_R = 0, \tag{6.4}$$

$$\left[\partial_\rho + \frac{W'}{W} + \frac{1}{a}\left(-m - \frac{1}{2}\omega + gA_\theta\right)\right]\varphi^{(m)}_L = 0, \tag{6.5}$$

with $\omega = 1 - a'$. In the patch surrounding $r = 0$, we can find the explicit solution to the above equations as

$$\begin{aligned}\varphi^{(m)}_L &= \frac{1}{W}\exp\left[\int^\rho d\rho'\frac{1}{a}\left(m + \frac{1}{2}\omega - gA_\theta\right)\right] \\ &= \frac{N_m}{W\sqrt{a}}\left(\frac{r}{r_0}\right)^{\frac{s}{2}} f_0^{\frac{1-t}{2}},\end{aligned} \tag{6.6}$$

with

$$\begin{aligned}s &= \frac{1}{\lambda}(1+2m) - \frac{g\xi_0}{\pi\lambda}, \\ t &= \frac{1}{\lambda}\left(m + \frac{1}{2} - n - \frac{g\xi_\infty}{2\pi}\right)\left(1 - \frac{r_0^2}{r_1^2}\right) + \frac{1}{\lambda}\left[n + \frac{g}{2\pi}(\xi_\infty - \xi_0)\right] + 1,\end{aligned} \tag{6.7}$$

where N_m is the normalization constant. We note that the solution for the right-handed gravitino is given by the one for the left-handed gravitino (6.6) with $(m, n, \xi_0, \xi_\infty)$ being replaced by $(-m, -n, -\xi_0, -\xi_\infty)$.

From the normalisation condition

$$\int d\theta \int d\rho\; Wa\;|\varphi^{(m)}_{L,R}|^2 < \infty, \tag{6.8}$$

we determine the normalisation constant of the general solution (6.6) as

$$N_m^2 = \frac{1}{2\pi r_0}\left(\int_0^\infty dx\frac{x^s}{(1+x^2)^t}\right)^{-1} \equiv \frac{\Gamma_m}{2\pi r_0}, \tag{6.9}$$

with

$$\Gamma_m \equiv \frac{2\Gamma[t]}{\Gamma[(1+s)/2]\Gamma[t-(1+s)/2]}. \tag{6.10}$$

[9] We will not be interested in the extra dimensional vector components of the gravitino ψ_m which are spin-$\frac{1}{2}$ components.

Then, in order for a left-handed zero mode to exist, the following normalisability conditions should be respected,

$$s > -1, \quad s - 2t < -1. \tag{6.11}$$

In terms of our original parameters, we require that

$$-\frac{1}{2}(1+\lambda) + \frac{g\xi_0}{2\pi} < m < n - \frac{1}{2}\left(1 - \lambda\frac{r_1^2}{r_0^2}\right) + \frac{g\xi_\infty}{2\pi}\,. \tag{6.12}$$

For the right-handed zero mode, the corresponding normalisability condition reads

$$n + \frac{1}{2}\left(1 - \lambda\frac{r_1^2}{r_0^2}\right) + \frac{g\xi_\infty}{2\pi} < m < \frac{1}{2}(1+\lambda) + \frac{g\xi_0}{2\pi}. \tag{6.13}$$

Using the relation between the FI term and the brane tension (3.15), as well as eqs. (4.10) and (4.11), the normalisability condition becomes for the left-handed mode

$$-\lambda < m < n\,, \tag{6.14}$$

and for the right-handed mode

$$n + 1 - \lambda\frac{r_1^2}{r_0^2} < m < 1. \tag{6.15}$$

If we compare the above calculation to the one of the non-SUSY brane tensions [23], we see that in the SUSY brane case, due to the localized FI terms, there are corrections to the gravitino wavefunction (6.6) and consequently to the normalisability conditions (6.12) and (6.13). Moreover, it is also expected that there are modifications to the KK massive modes of the gravitino [23].

For the "football"-shaped solutions, we have that $q = 4g$ and $\lambda = n$. For $n = 1$, we obtain the well-known Salam-Sezgin vacuum with one 4D chiral gravitino, the left-handed zero mode $\varphi_L^{(0)}$. For $n > 1$, we see that we will always have normalisable left-handed zero modes $\varphi_L^{(m)}$, but no right-handed ones. The action of the Z_2 parity on the left-handed modes requires that m is even. Therefore, for n even, $(n-1)$ left-handed zero modes are allowed, and for n odd, n left-handed zero modes survive. In all latter cases, $\mathcal{N} = 1$ SUSY is preserved by the background.

It would be surprising to find that for $n > 2$, the $\mathcal{N} = 1$ unwarped solutions support more than one 4D chiral gravitinos, because one would expect only one surviving in $\mathcal{N} = 1$ 4D effective supergravity. The mass terms for these chiral gravitinos would be forbidden due to the U(1) gauge symmetries: one is the $\mathrm{U}(1)_Q$ isometry of the axisymmetric extra dimensions and the other is the $\mathrm{U}(1)_R$ gauge symmetry.[10] The charge operator $\hat{Q}$ of the

[10] Both of them can be anomaly-free due to the generalised Green-Schwarz mechanism where the U(1) gauge bosons get masses but the theory is still invariant due to the axionic coupling to the gauge boson. The gauge boson mass of the $\mathrm{U}(1)_Q$ could be read from a possible gravitational Chern-Simons term in the three form field strength, which arises due to the supersymmetric completion of the Green-Schwarz term [39], as in the case of the $\mathrm{U}(1)_R$ gauge boson. The computation of it, is beyond the scope of the present paper.

$\mathrm{U}(1)_Q$ commutes with the 6D Dirac mass operator [40] and it is given in the 6D spinor basis by

$$\hat{Q} = -i\partial_\theta + \frac{1}{2}\sigma^3 \otimes \gamma^5. \tag{6.16}$$

Let us now consider the 4D effective action for the left-handed zero modes of the gravitino coupled to two $\mathrm{U}(1)$ gauge bosons. The part of the effective low energy Lagrangian that is relevant in our discussion, is similar with the non-SUSY bulk model [41], and reads

$$\begin{aligned}\mathcal{L}_{\rm eff} &= -\frac{1}{4}F^2_{\mu\nu} - \frac{1}{4}F'^2_{\mu\nu} \\ &\quad + \sum_m \tilde{\psi}^{(m)\dagger}_{\mu L}\bar{\sigma}^{[\mu}\sigma^\nu\bar{\sigma}^{\lambda]}\left(\partial_\nu + \frac{1}{4}\omega_{\nu\alpha\beta}\sigma^{[\alpha}\bar{\sigma}^{\beta]} - ig_4 R A_\nu - ig'_4 Q A'_\nu\right)\tilde{\psi}^{(m)}_{\lambda L}\end{aligned} \tag{6.17}$$

where A_μ, A'_μ are the $\mathrm{U}(1)_R$ and $\mathrm{U}(1)_Q$ gauge bosons with the 4D effective gauge couplings g_4 and g'_4, respectively. Here, we note that the R and Q charge operators take the values $+1$ and $m + \frac{1}{2}$ for $\tilde{\psi}^{(m)}_{\mu L}$, respectively. Then, after changing the basis of the gauge bosons to $A_{1\mu}$ and $A_{2\mu}$ as

$$A_{1\mu} = \frac{1}{\sqrt{4g_4^2 + g'^2_4}}\left(g'_4 A_\mu - 2g_4 A'_\mu\right), \tag{6.18}$$

$$A_{2\mu} = \frac{1}{\sqrt{4g_4^2 + g'^2_4}}\left(2g_4 A_\mu + g'_4 A'_\mu\right), \tag{6.19}$$

the above action is rewritten as

$$\begin{aligned}\mathcal{L}_{\rm eff} &= -\frac{1}{4}F^2_{1\mu\nu} - \frac{1}{4}F^2_{2\mu\nu} \\ &\quad + \sum_m \tilde{\psi}^{(m)\dagger}_{\mu L}\bar{\sigma}^{[\mu}\sigma^\nu\bar{\sigma}^{\lambda]}\left(\partial_\nu + \frac{1}{4}\omega_{\nu\alpha\beta}\sigma^{[\alpha}\bar{\sigma}^{\beta]} - ig_1 Q_1 A_{1\nu} - ig_2 Q_2 A_{2\nu}\right)\tilde{\psi}^{(m)}_{\lambda L},\end{aligned} \tag{6.20}$$

where the new charge operators are

$$Q_1 = R - 2Q, \quad Q_2 = \frac{2g_4^2}{g'^2_4}R + Q, \tag{6.21}$$

and the new gauge couplings are

$$g_1 = \frac{g_4 g'_4}{\sqrt{4g_4^2 + g'^2_4}}, \quad g_2 = \frac{g'^2_4}{\sqrt{4g_4^2 + g'^2_4}}. \tag{6.22}$$

In this case, we note that the Q_1 charge of the left-handed zero mode with m winding number is $Q_1 = -2m$.

Let us now suppose that at low energies, only Q_1 survives while Q_2 is broken.[11] Then, for the "football" solutions, after the Z_2 projection, the remaining left-handed zero modes with nonzero even and opposite m or Q_1 charges can be paired up to make a 4D Dirac spinor

$$\Psi^{(m)}_\mu = \left(\tilde{\psi}^{(m)}_{\mu L}, -i\sigma^2\tilde{\psi}^{(-m)*}_{\mu L}\right)^T, \tag{6.23}$$

[11] If a linear combination Q_2 is anomalous, it could be broken due to the corresponding FI terms without breaking SUSY.

so that they get coupled by their Dirac masses. Therefore, there can be only one chiral massless mode of the gravitino with $m = 0$, *i.e.,* the zero mode uncharged under the $\mathrm{U}(1)_1$. The above mechanism for pairing the left-handed modes, relies on the VEV of a complex scalar field that breaks the $\mathrm{U}(1)_2$, with appropriate quantum numbers which makes a Yukawa coupling with the left-handed modes Q_2-invariant. If in addition we write down localised Majorana mass terms on regularised branes [23] for the chiral $m = 0$ massless mode, we can end up with a non-zero mass 4D Majorana gravitino. In this case, the remaining $\mathcal{N} = 1$ SUSY should be also broken by nonzero F-terms on the branes.

For the general warped solution, we find that there are multiple zero modes of left-handed gravitino with even m while there could also exist zero modes of right-handed gravitino with odd m. In this case, the number of zero modes depends on the warping and the monopole number.

In the presence of the localised FI terms, for a spin-$\frac{1}{2}$ fermion with the same $\mathrm{U}(1)_R$ charge as the gravitino, a similar analysis can be done like in ref. [21]. There is a difference from the gravitino case only by the warp factor dependence of the wavefunction. The wavefunction of the zero mode is given by eq. (6.6) with W being replaced by W^2. However, for the spin-$\frac{1}{2}$ fermion, the weighting function in the norm integration (6.8) is changed to $W^3 a$, so the normalization condition is the same as eqs. (6.14) and (6.15) in the gravitino case. Therefore, a spin-$\frac{1}{2}$ fermion has the same spectrum as the one of the gravitino. Thus, a pair of the spin-$\frac{1}{2}$ zero modes with $(m, -m)$ could be regarded as being eaten by a pair of the zero modes of the gravitino with $(m, -m)$ to make up a massive 4D Dirac gravitino. Consequently, each massive 4D Dirac gravitino should be part of an $\mathcal{N} = 1$ massive spin-$\frac{3}{2}$ supermultiplet.

7. Conclusions

In this work, we examined the way to supersymmetrise the Salam-Sezgin model in the presence of codimension-2 branes carrying only tension. We have modified the brane action by adding brane localised FI terms and in addition changed the SUSY transformations where necessary. The resulting brane action respects $\mathcal{N} = 1$ SUSY, if the FI terms are chosen appropriately (related to the brane tension) and requires the presence of a Z_2 symmetry to be realised.

The axisymmetric background solution for the above system is the same for the metric and dilaton fields as for the non-SUSY brane action system [10 − 12]. However, the gauge field solution acquires an additional Wilson line contribution. The last is important when discussing the SUSY of the background solution. There, we find that the unwarped solution with "football"-shaped internal space always respects $\mathcal{N} = 1$ SUSY, in contrast with the non-SUSY brane action system.

The gravitino zero mode equation of motion was then analysed for the above-mentioned background. We found the conditions for which left- and right-handed modes are normalisable. We have focused on the unwarped "football" background case and remarked that always a left-handed mode survives with zero winding number m. For $n > 3$ there are additional chiral zero modes with non-zero m. It is conceivable that these extra modes, is some cases, can be paired to Dirac four-dimensional spinors, leaving only one chiral zero mode in the massless spectrum.

A natural continuation of the present study is to include $\mathcal{N}=1$ matter multiplets (chiral and vector) on the branes with couplings to the bulk fields. This would require a regularisation of the brane, *e.g.,* in the lines of [27], since the brane source terms coupled to the bulk fields other than the brane tension would lead to classical divergences. Then, it is expected that SUSY will completely fix the couplings of the brane with the bulk fields. In this way, we can reconsider the issue of moduli stabilisation [39, 42, 30] in the specific gauged supergravity with the supersymmetric branes. Moreover, if the MSSM fields are localised on one of the branes, one is expected to draw important conclusions about the supersymmetry breaking transmission between the bulk and the branes, or between the two distant branes in the different geometry than a torus. A generalization of the above study to multibrane systems without the axial symmetry [13] could also be interesting in that respect.

In addition, a necessary work that is important to be done is the consistency check of our proposal to eliminate the chiral modes of the gravitino with non-zero winding number m. One should study whether it is possible in the specific model to have one of the two U(1)'s naturally much heavier than the other, thus leaving one gravitino with a small mass in the low energy spectum. Moreover, the decoupling of the chiral modes with non-zero m relies on the nonzero VEV of a scalar field which has a right quantum number Q_2 for the Yukawa coupling. We plan to investigate the above questions in the near future.

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A. Notations and conventions

We use the metric signature $(-,+,+,+,+,+)$ for the 6D metric. The index conventions are the following: (1) for the Einstein indices we use $M,N,\ldots=0,\ldots,5,6$ for the 6D indices, $\mu,\nu,\ldots,=0,\ldots,3$ for the 4D indices and $m,n,\ldots=5,6$ for the internal 2D indices, (2) for the Lorentz indices we use $A,B,\ldots=0,\ldots,5,6$ for the 6D indices, $\alpha,\beta,\ldots=0,\ldots,3$ for the 4D indices and $a,b,\ldots=5,6$ for the internal 2D indices.

We take the gamma matrices in the locally flat coordinates [9], satisfying $\{\Gamma_A,\Gamma_B\}=2\eta_{\rm AB}$, to be

$$\Gamma_\alpha=\sigma^1\otimes\gamma_\alpha,\quad \Gamma_5=\sigma^1\otimes\gamma_5,\quad \Gamma_6=\sigma^2\otimes\mathbf{1}, \tag{A.1}$$

where γ's are the 4D gamma matrices with $\gamma_5^2 = 1$ and σ's are the Pauli matrices with $[\sigma^i, \sigma^j] = 2i\epsilon_{ijk}\sigma^k$, with $i, j, k = 1, 2, 3$,

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A.2}$$

The curved gamma matrices on the other hand are given in terms of the ones in the locally flat coordinates as $\Gamma^M = e_A^{\ M}\Gamma^A$ where $e_A^{\ M}$ is the 6D vielbein. In addition, the 6D chirality operator is given by

$$\Gamma_7 = \Gamma_0\Gamma_1 \cdots \Gamma_6 = \sigma^3 \otimes \mathbf{1}. \tag{A.3}$$

The convention for 4D gamma matrices is that

$$\gamma^\alpha = \begin{pmatrix} 0 & \sigma^\alpha \\ \bar{\sigma}^\alpha & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}, \tag{A.4}$$

with $\sigma^\alpha = (\mathbf{1}, \sigma^i)$ and $\bar{\sigma}^\alpha = (-\mathbf{1}, \sigma^i)$. The chirality projection operators are defined as $P_L = (1 + \gamma^5)/2$ and $P_R = (1 - \gamma^5)/2$.

Finally, some useful quantities which we use in the text are the following

$$\Gamma^{\alpha 5} = \mathbf{1} \otimes \gamma^\alpha\gamma^5, \quad \Gamma^{\alpha 6} = i\sigma^3 \otimes \gamma^\alpha, \quad \Gamma^{56} = i\sigma^3 \otimes \gamma^5. \tag{A.5}$$

References

[1] N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *The hierarchy problem and new dimensions at a millimeter*, *Phys. Lett.* **B 429** (1998) 263 [hep-ph/9803315];
I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *New dimensions at a millimeter to a Fermi and superstrings at a TeV*, *Phys. Lett.* **B 436** (1998) 257 [hep-ph/9804398];
L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221].

[2] L. Randall and R. Sundrum, *Out of this world supersymmetry breaking*, *Nucl. Phys.* **B 557** (1999) 79 [hep-th/9810155].

[3] M.A. Luty and R. Sundrum, *Radius stabilization and anomaly-mediated supersymmetry breaking*, *Phys. Rev.* **D 62** (2000) 035008 [hep-th/9910202].

[4] G.F. Giudice, M.A. Luty, H. Murayama and R. Rattazzi, *Gaugino mass without singlets*, *JHEP* **12** (1998) 027 [hep-ph/9810442].

[5] A. Anisimov, M. Dine, M. Graesser and S.D. Thomas, *Brane world SUSY breaking*, *Phys. Rev.* **D 65** (2002) 105011 [hep-th/0111235]; *Brane world SUSY breaking from string/M theory*, *JHEP* **03** (2002) 036 [hep-th/0201256].

[6] A. Falkowski, H.M. Lee and C. Ludeling, *Gravity mediated supersymmetry breaking in six dimensions*, *JHEP* **10** (2005) 090 [hep-th/0504091].

[7] M. Schmaltz and R. Sundrum, *Conformal sequestering simplified*, *JHEP* **11** (2006) 011 [hep-th/0608051];
S. Kachru, L. McAllister and R. Sundrum, *Sequestering in string theory*, *JHEP* **10** (2007) 013 [hep-th/0703105].

JHEP01(2008)008

[8] A. Falkowski, Z. Lalak and S. Pokorski, *Supersymmetrizing branes with bulk in five-dimensional supergravity*, *Phys. Lett.* **B 491** (2000) 172 [hep-th/0004093];
J. Bagger, D. Nemeschansky and R.-J. Zhang, *Supersymmetric radion in the Randall-Sundrum scenario*, *JHEP* **08** (2001) 057 [hep-th/0012163];
E. Bergshoeff, R. Kallosh and A. Van Proeyen, *Supersymmetry in singular spaces*, *JHEP* **10** (2000) 033 [hep-th/0007044];
J. Bagger and D.V. Belyaev, *Supersymmetric branes with (almost) arbitrary tensions*, *Phys. Rev.* **D 67** (2003) 025004 [hep-th/0206024].

[9] H. Nishino and E. Sezgin, *Matter and gauge couplings of $N = 2$ supergravity in six-dimensions*, *Phys. Lett.* **B 144** (1984) 187;
A. Salam and E. Sezgin, *Chiral compactification on Minkowski X S^2 of $N = 2$ Einstein-Maxwell supergravity in six-dimensions*, *Phys. Lett.* **B 147** (1984) 47.

[10] G.W. Gibbons, R. Guven and C.N. Pope, *3-branes and uniqueness of the Salam-Sezgin vacuum*, *Phys. Lett.* **B 595** (2004) 498 [hep-th/0307238].

[11] Y. Aghababaie et al., *Warped brane worlds in six dimensional supergravity*, *JHEP* **09** (2003) 037 [hep-th/0308064].

[12] C.P. Burgess, F. Quevedo, G. Tasinato and I. Zavala, *General axisymmetric solutions and self-tuning in 6D chiral gauged supergravity*, *JHEP* **11** (2004) 069 [hep-th/0408109].

[13] H.M. Lee and C. Ludeling, *The general warped solution with conical branes in six-dimensional supergravity*, *JHEP* **01** (2006) 062 [hep-th/0510026].

[14] S.L. Parameswaran, G. Tasinato and I. Zavala, *The 6D superswirl*, *Nucl. Phys.* **B 737** (2006) 49 [hep-th/0509061].

[15] S.M. Carroll and M.M. Guica, *Sidestepping the cosmological constant with football-shaped extra dimensions*, hep-th/0302067;
I. Navarro, *Codimension two compactifications and the cosmological constant problem*, *JCAP* **09** (2003) 004 [hep-th/0302129];
Y. Aghababaie, C.P. Burgess, S.L. Parameswaran and F. Quevedo, *Towards a naturally small cosmological constant from branes in 6D supergravity*, *Nucl. Phys.* **B 680** (2004) 389 [hep-th/0304256].

[16] I. Navarro, *Spheres, deficit angles and the cosmological constant*, *Class. and Quant. Grav.* **20** (2003) 3603 [hep-th/0305014];
H.-P. Nilles, A. Papazoglou and G. Tasinato, *Selftuning and its footprints*, *Nucl. Phys.* **B 677** (2004) 405 [hep-th/0309042];
J. Garriga and M. Porrati, *Football shaped extra dimensions and the absence of self-tuning*, *JHEP* **08** (2004) 028 [hep-th/0406158].

[17] H.M. Lee, *A comment on the self-tuning of cosmological constant with deficit angle on a sphere*, *Phys. Lett.* **B 587** (2004) 117 [hep-th/0309050].

[18] K. Koyama, *The cosmological constant and dark energy in braneworlds*, arXiv:0706.1557.

[19] S. Mukohyama, Y. Sendouda, H. Yoshiguchi and S. Kinoshita, *Warped flux compactification and brane gravity*, *JCAP* **07** (2005) 013 [hep-th/0506050];
H. Yoshiguchi, S. Mukohyama, Y. Sendouda and S. Kinoshita, *Dynamical stability of six-dimensional warped flux compactification*, *JCAP* **03** (2006) 018 [hep-th/0512212].

[20] H.M. Lee and A. Papazoglou, *Scalar mode analysis of the warped Salam-Sezgin model*, *Nucl. Phys.* **B 747** (2006) 294 [*Erratum ibid.* **B 765** (2007) 200] [hep-th/0602208];
S.L. Parameswaran, S. Randjbar-Daemi and A. Salvio, *Gauge fields, fermions and mass gaps in* 6*D brane worlds*, *Nucl. Phys.* **B 767** (2007) 54 [hep-th/0608074];
C.P. Burgess, C. de Rham, D. Hoover, D. Mason and A.J. Tolley, *Kicking the rugby ball: perturbations of* 6*D gauged chiral supergravity*, *JCAP* **02** (2007) 009 [hep-th/0610078].

[21] S.L. Parameswaran, S. Randjbar-Daemi and A. Salvio, *Stability and negative tensions in* 6*D brane worlds*, arXiv:0706.1893.

[22] T. Kobayashi and M. Minamitsuji, *Gravity on an extended brane in six-dimensional warped flux compactifications*, *Phys. Rev.* **D 75** (2007) 104013 [hep-th/0703029].

[23] H.M. Lee and A. Papazoglou, *Gravitino in six-dimensional warped supergravity*, *Nucl. Phys.* **B 792** (2008) 166 [arXiv:0705.1157].

[24] A.J. Tolley, C.P. Burgess, D. Hoover and Y. Aghababaie, *Bulk singularities and the effective cosmological constant for higher co-dimension branes*, *JHEP* **03** (2006) 091 [hep-th/0512218];
A.J. Tolley, C.P. Burgess, C. de Rham and D. Hoover, *Scaling solutions to* 6*D gauged chiral supergravity*, *New J. Phys.* **8** (2006) 324 [hep-th/0608083];
B. Himmetoglu and M. Peloso, *Isolated Minkowski vacua and stability analysis for an extended brane in the rugby ball*, *Nucl. Phys.* **B 773** (2007) 84 [hep-th/0612140];
T. Kobayashi and M. Minamitsuji, *Brane cosmological solutions in six-dimensional warped flux compactifications*, *JCAP* **07** (2007) 016 [arXiv:0705.3500];
E.J. Copeland and O. Seto, *Dynamical solutions of warped six dimensional supergravity*, *JHEP* **08** (2007) 001 [arXiv:0705.4169].

[25] K.-i. Maeda and H. Nishino, *Cosmological solutions in* $D = 6$, $N = 2$ *Kaluza-Klein supergravity: Friedmann universe without fine tuning*, *Phys. Lett.* **B 154** (1985) 358; *Attractor universe in six-dimensional* $N = 2$ *supergravity Kaluza-Klein theory*, *Phys. Lett.* **B 158** (1985) 381.

[26] B. Carter, R.A. Battye and J.-P. Uzan, *Gradient formula for linearly self-interacting branes*, *Commun. Math. Phys.* **235** (2003) 289 [hep-th/0204042];
M. Kolanovic, M. Porrati and J.-W. Rombouts, *Regularization of brane induced gravity*, *Phys. Rev.* **D 68** (2003) 064018 [hep-th/0304148];
S. Kanno and J. Soda, *Quasi-thick codimension* 2 *braneworld*, *JCAP* **07** (2004) 002 [hep-th/0404207];
I. Navarro and J. Santiago, *Gravity on codimension* 2 *brane worlds*, *JHEP* **02** (2005) 007 [hep-th/0411250];
C. de Rham, *The effective field theory of codimension-two branes*, arXiv:0707.0884.

[27] M. Peloso, L. Sorbo and G. Tasinato, *Standard* 4*D gravity on a brane in six dimensional flux compactifications*, *Phys. Rev.* **D 73** (2006) 104025 [hep-th/0603026];
E. Papantonopoulos, A. Papazoglou and V. Zamarias, *Regularization of conical singularities in warped six-dimensional compactifications*, *JHEP* **03** (2007) 002 [hep-th/0611311];
N. Kaloper and D. Kiley, *Charting the landscape of modified gravity*, *JHEP* **05** (2007) 045 [hep-th/0703190].

[28] J. Vinet and J.M. Cline, *Can codimension-two branes solve the cosmological constant problem?*, *Phys. Rev.* **D 70** (2004) 083514 [hep-th/0406141]; *Codimension-two branes in*

six-dimensional supergravity and the cosmological constant problem, *Phys. Rev.* **D 71** (2005) 064011 [hep-th/0501098].

[29] E. Papantonopoulos, A. Papazoglou and V. Zamarias, *Induced cosmology on a regularized brane in six-dimensional flux compactification*, arXiv:0707.1396;
M. Minamitsuji and D. Langlois, *Cosmological evolution of regularized branes in 6D warped flux compactifications*, *Phys. Rev.* **D 76** (2007) 084031 [arXiv:0707.1426].

[30] C.P. Burgess, D. Hoover and G. Tasinato, *UV caps and modulus stabilization for 6D gauged chiral supergravity*, *JHEP* **09** (2007) 124 [arXiv:0705.3212].

[31] E. Elizalde, M. Minamitsuji and W. Naylor, *Casimir effect in rugby-ball type flux compactifications*, *Phys. Rev.* **D 75** (2007) 064032 [hep-th/0702098];
M. Minamitsuji, *Casimir effect in a 6D warped flux compactification model*, arXiv:0704.3623.

[32] S. Fujii, T. Kobayashi and T. Shiromizu, *Low energy effective theory on a regularized brane in six-dimensional flux compactifications*, *Phys. Rev.* **D 76** (2007) 104052 [arXiv:0708.2534];
F. Arroja, T. Kobayashi, K. Koyama and T. Shiromizu, *Low energy effective theory on a regularized brane in 6D gauged chiral supergravity*, *JCAP* **12** (2007) 006 [arXiv:0710.2539].

[33] A.J. Tolley, C.P. Burgess, C. de Rham and D. Hoover, *Exact wave solutions to 6D gauged chiral supergravity*, arXiv:0710.3769.

[34] H.M. Lee, H.P. Nilles and M. Zucker, *Spontaneous localization of bulk fields: the six-dimensional case*, *Nucl. Phys.* **B 680** (2004) 177 [hep-th/0309195].

[35] E.A. Mirabelli and M.E. Peskin, *Transmission of supersymmetry breaking from a 4-dimensional boundary*, *Phys. Rev.* **D 58** (1998) 065002 [hep-th/9712214];
H.M. Lee, *Softness of supersymmetry breaking on the orbifold $T^2/\mathbb{Z}_2$*, *JHEP* **06** (2005) 044 [hep-th/0502093].

[36] A. Falkowski, *On the one-loop Kähler potential in five-dimensional brane-world supergravity*, *JHEP* **05** (2005) 073 [hep-th/0502072].

[37] S. Randjbar-Daemi, A. Salam, E. Sezgin and J.A. Strathdee, *An anomaly free model in six-dimensions*, *Phys. Lett.* **B 151** (1985) 351.

[38] S.D. Avramis, A. Kehagias and S. Randjbar-Daemi, *A new anomaly-free gauged supergravity in six dimensions*, *JHEP* **05** (2005) 057 [hep-th/0504033];
S.D. Avramis and A. Kehagias, *A systematic search for anomaly-free supergravities in six dimensions*, *JHEP* **10** (2005) 052 [hep-th/0508172];
R. Suzuki and Y. Tachikawa, *More anomaly-free models of six-dimensional gauged supergravity*, *J. Math. Phys.* **47** (2006) 062302 [hep-th/0512019].

[39] Y. Aghababaie, C.P. Burgess, S.L. Parameswaran and F. Quevedo, *SUSY breaking and moduli stabilization from fluxes in gauged 6D supergravity*, *JHEP* **03** (2003) 032 [hep-th/0212091].

[40] C. Wetterich, *Chiral fermions in six-dimensional gravity*, *Nucl. Phys.* **B 253** (1985) 366;
J.-M. Schwindt and C. Wetterich, *Holographic branes*, *Phys. Lett.* **B 578** (2004) 409 [hep-th/0309065].

[41] J.-M. Schwindt and C. Wetterich, *Dark energy cosmologies for codimension-two branes*, *Nucl. Phys.* **B 726** (2005) 75 [hep-th/0501049].

[42] A.P. Braun, A. Hebecker and M. Trapletti, *Flux stabilization in* 6 *dimensions: D-terms and loop corrections*, *JHEP* **02** (2007) 015 [hep-th/0611102].